617 LEC Sequences and Codes

Final Project

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Mathematization of mmWave Radar

1. Introduction

The mmWave product line from Texas Instruments offers a high resolution, low power radar solution for a wide range of applications including medical, industrial, and automotive. This family of radar parts has a transmitter bandwidth of 4GHz, but is limited by a hardware front-end which creates an IF signal limited to 10MHz bandwidth. The fundamental received signal is not directly available for mixing, or autocorrelation operations. An in-depth review of the processing chain may reveal additional opportunities for decreasing the receive noise.

2. Overview

The TI mmWave 6843AOP radar sensor is a popular component which offers both an ARM core and C6xxx DSP in a single package with built-in radar front end an on-package antennas. The radar front-end uses a 20GHz synth to generate a 60 GHz transmission frequency. This signal is sent to the transmitters and mixed with the receive signal to form the intermediate frequency (IF). Because the two signals are mixed, only receive signals detected during the outgoing transmission and within the allowed IF bandwidth will form detectable signals.

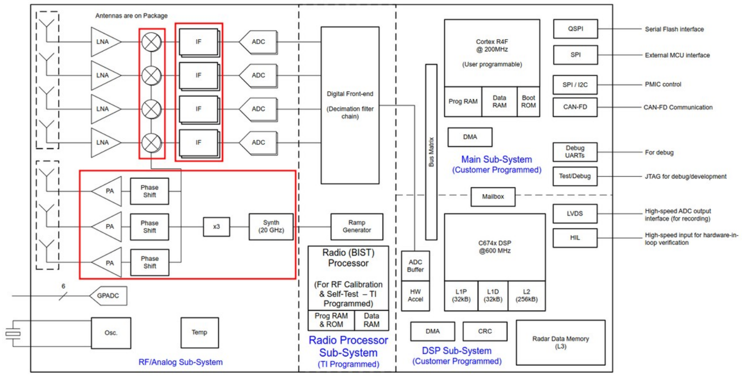

Figure 1: IWR6843AOP Block Diagram

For a given chirp signal in the transmitted sequence of chirps C_i, the receive signal will contain multiple detected reflections as well as noise and will take the following form.

$$\Sigma_n C_i(Z+\alpha_n)+N \tag{1}$$

This received signal is combined with the transmitted signal to create an Intermediate Frequency (IF) signal which will include the mixer products of the transmitted and received signal.

This signal is then directly sampled for signal limiting the bandwidth of the signal to half the sampling frequency and restricting the signal to the frequency difference component. The full data set collected for the standard processing chain contains ADC samples collected for each receive antenna for the duration of the chirp. The FFT of these fast-time samples produce a complex FFT output which represents range and phase information. A tunable gap is present between chirps to allow the LO to reset. This gap is also used to adjust the Doppler resolution for the expected target velocity. For subsequent chirps, the peaks in the range FFT correlate, but the output phase is shifted due to the change in position of the target.

3. Signal to Noise Ratio

The signal reflection from the target is consistently detected while the noise is stochastic. In order to improve the signal to noise ratio, multiple transmissions can be convolved. Results from Zhang et al[1] suggest that this method is relevant given a stationary target and properly selected chirps. Zhang et al recommends using unique chirps which do not resample the same noise. Range resolution is given by Equation 2.

$$Range\ Resolution = \frac{c}{2B} \tag{2}$$

In order to ensure the data can be directly combined, the chirps must have the same range resolution so received peaks appear in the same location. Using the same bandwidth with a different starting frequency will resample the same received data without repeating the measurement of noise. Repeating multiple transmission quickly to improve SNR for a reliable range measurement will improve range results which can then be applied to slow-chirp Doppler results.

4. Impact of Doppler Shift

Doppler shift caused by a two way transmission is given by Equation 11. For a typical chirp configuration used for an indoor industrial application will have a chirp repetition period around 60 us[1]. This chirp configuration has a maximum velocity of 7.2 m/s before phase change between subsequent chirp samples becomes ambiguous.

$$\Delta f = \left(1 - \frac{c - 7\,m/s}{c + 7\,m/s}\right) 60\,GHz = 2.8\ \text{kHz} \tag{3}$$

In this example, an object moving at the maximum velocity of 7.2 m/s will not cause a Doppler shift capable of impacting the range measurement. Given a 10MHz IF bandwidth, an FFT output with 256 bins will have output range bins with 39kHz bandwidth. A frequency change of 39kHz represents a target moving at 97 m/s which is very high for most industrial applications. Other chirp configurations or applications may experience significant Doppler shift of 1-2 range bins that must be compensated for.

1 Estimate of 60us chirp period taken from default chirp configurations recommendations provided with IWR6843AOP from Texas Instruments `mmWave Sensing Estimator` toolbox.

5. Energy Conservation in Doppler

Considering the energy and momentum of the target, as well as the energy and momentum of a single photon, we are able to write equations for the initial and final energy in the system to derive the classical equation for Doppler shift. The energy and momentum of the radar signal of a photon is only considered after it has left the radar station and before it has returned as shown in Figure 2.

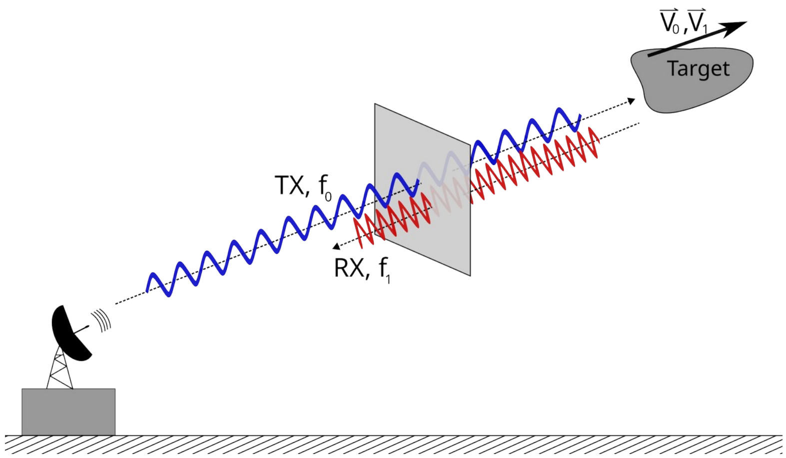

Figure 2. Model of Radar Reflection

A radar station transmits a signal at an initial frequency f_0 striking a target with mass m, and an initial velocity v_0. The signal experiences a Doppler shift due to the movement of the target and is reflected with a frequency f_1. The target continues on with a new velocity v_1. Considering the system constructed by the incoming and outgoing photon and the target, we construct a pair of equations representing conservation of Energy and conservation of Momentum with positive momentum defined in the direction of the original wave transmission.

Sum of Energy $$\Sigma E = E_i = E_f, \quad nhf_0 + \frac{1}{2} m v_0^2 = nhf_1 + \frac{1}{2} m v_1^2 \tag{4}$$

Sum of Momentum $$\Sigma p = p_i = p_f, \quad \frac{1}{c} nhf_0 + m v_0 = -\frac{1}{c} nhf_1 + m v_1 \tag{5}$$

These expressions can be rearranged to group frequency and velocity terms.

Statement from Energy	$nh(f_0 - f_1) = \frac{1}{2} m (v_1^2 - v_0^2)$	(6)
Statement from Momentum	$nh(f_0 + f_1) = (v_1 - v_0) mc$	(7)

Equations 6 and 7 can then be simplified by substituting $v_1 = v_0 + \Delta v$ and removing Δv^2 terms because, for non-relativistic speeds, this term is negligible.

$$nh(f_0 - f_1) = m \Delta v\, v_0 \tag{8}$$

$$nh(f_0 + f\,1) = m \Delta v\, c \tag{9}$$

Finally relating the energy and momentum statements by dividing equation 8 by equation 9 yields an expression which only includes the desired values,

$$\frac{f_0 - f_1}{f_0 + f_1} = \frac{v_0}{c} \tag{10}$$

Gathering frequency terms and solving for f_1 yields the non-relativistic two-way Doppler equation.

$$f_1 = f_0 \frac{c - v_0}{c + v_0} \tag{11}$$

This demonstrates that for objects with no initial velocity, $v_0 = 0$, there will be no frequency change. For objects moving away from the transmitter, there will be a negative Doppler shift, $\Delta f < 0$ and for objects moving towards the transmitter, there will be a positive Doppler shift, $\Delta f > 0$. Which is consistent with our expectations.

Bibliography

1: J. Zhang, Y. Li, W. Li, Z. Zhang, C. Gu and J. Mao, Multi-Chirps Convolution Technique for Range Spectrum Signal Enhancement Based on a 60-GHz MIMO FMCW Radar, 2023